1 Introduction

Depending on your background, some of these exercises may be easy, while others may be more difficult. We simply ask you to do your best with these problems, within the limits of the time allocated for the course.

Read **these reminders** before you begin!

(keeping track)

(EXPLAIN UNITS)

Explain units and unit conversions (possibly also including map scale conversions). Try to find a short, clear and readable explanation. Write as you like, with an example or with a more general explanation. It is all up to you what you are able to do and how you want to explain. Also think generally about what an "explanation" is. Is there a difference between describing and defining?

(ESTIMATION)

a) How many iPhones are sold in Sweden every year? (in order to practice your thinking, don't google anything - instead make an estimate based on whatever knowledge you may already have).

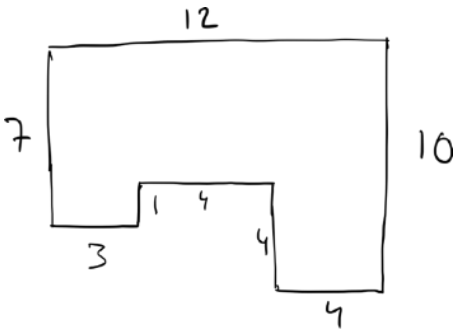
b) If a horse with the height of h1 weighs w1 kg, what would you expect a horse with the height h2 to weigh?

(INTERPRETING QUANTITATIVE INFORMATION)

Carefully consider the following statements. We want you to consider the nature of the statements as you can understand them, and are not asking you to google facts.

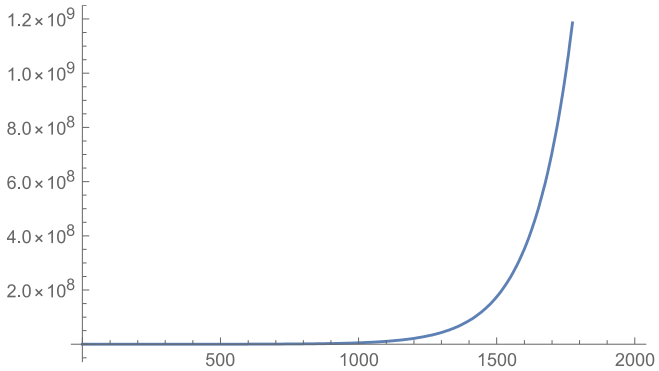
- 843 pupils, of which 432 girls and 421 boys.
- Based on yearly weather statistics, we can statistically conclude that there is a 15% chance of rain on any day of the year.
- It makes no difference what we do here in Sweden, because we are only responsible for 0.2% of greenhouse gases globally.
- The city could easily afford to employ more unemployed people, since if we employ them, we also get more taxpayers.

- After rolling a die 100 times ‘6’ appears 20 times, therefore the die is loaded.
- Is there anything wrong or misleading with some of these statements and graphs? How? Can statements be wrong in different ways? Can you group them in some way?
- Consider this house plan!



- For every shirt you buy we will give 2 cents to tree plantations.
- Since almost 100% of violent crime is perpetrated by men, all men should take responsibility.
- If we raise the income tax from 40% to 60% we would increase the income tax revenues by 50%.
- An investigation has shown that if you only eat organic food, the levels of pesticides in your body decreases dramatically, on the average by 72% after only two weeks. (claim used in marketing of organic foods)
- Consider this world map!

- It will obviously take time, but our long term goal is for everyone to have their own personal assistant.
- In the last years, Sweden has received 20 million refugees.
- Last Saturday, the King of Sweden died in an accident.
- 99% fat free milk! (as written on milk cartons in North America)
- We can see that the world population started to really take off from approximately 14-1500.



(investigating the abstract)

(SQUARE ROOT ALGORITHM)

The following iterative algorithm for calculating the square root was known already to the Babylonians:

$$x_{n+1} = \frac{x_n + \frac{a}{x_n}}{2}$$

a) Try it out on a couple of numbers and show an example. (a is the number we want to calculate the square root of). You can use **sqrt.py**. If you do not have Python installed, you can use the Online Python link.

b) (voluntary) Make an attempt to explain why the algorithm works! Would the algorithm work for all initial values?

(PROVE ALGEBRAIC LAWS)

a) Prove the algebraic laws ab=ba and (a+b)^2=a^2+2ab+b^2.

b) (voluntary) Try to explain why (-1)^2 = 1.

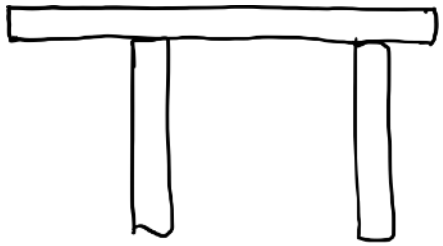
(SQRT(2) IS IRRATIONAL)

Read and understand **this proof**. Consider how it feels when you understand the proof, compared to just before you understood it. Explain the main idea of the proof. (there are many links about this on the web, feel free to have a look if you like!)

(investigating the world)

(BEAM ON TWO SUPPORTS)

You have a beam on two supports, see the figure. Imagine that the beam and the supports can be moved sideways. What can you say about the weight on each support?



(SOUND INTENSITY)

(voluntary) How much does the sound intensity decrease with the distance from the source? Motivate your answer.

(design)

(ROTARY ENCODER)

For a rotary sensor, we can construct hardware with electrical connectors so that a binary number indicates one out of many possible angle measurements. However, in the transitions between adjacent sectors, it would be very desirable if only one bit changes, to prevent spurious erratic readings of angles that are nowhere close. Try – if possible - to construct a binary code so that this condition is satisfied!

(thinking)

(REASONING – AMBIGUITY AND INTERPRETATION)

This is to raise your awareness of the imprecise nature of most communication. You must be aware of this and *as well as possible interpret to your own precise understanding*. The same holds for the problems in this course!

Briefly explain the imprecision in each of the following statements:

- July is a summer month
- You will receive the payment if you meet the deadline.
- You can choose A and B, but not C.
- “Is there anything behind the car?” “I don’t see anything.”

- The temperature is high.
- We want you to design a better solution.

(REASONING - WHAT IS KNOWN AND NOT?)

This kind of abstract thinking is important for being able to plan what you need in order to determine something, and to understand what you can determine from what you know. (Briefly write out each question for clarity for the reader)

- You have two sides and an angle. Is the triangle "known" (= unambiguously determined)? Does it matter which angle?
- You know the value of x^2-bx where b is a known number. Is x known?
- You know a-b and a*b. Is a and b known?
- You have a point in the plane. Is the line that passes through this point known?
- You know the shape of a ground floor plan for a building and the length of one side. Is the area known?
- There are 26 sheep and 10 goats on a ship. How old is the captain?
- You have a graph where each edge has a known length. Is then the shortest path from node A to another node B known? What if some "lengths" are negative?
- You don't know x. Can you say anything about the sign of x^2-x?

Consider every question, but you may answer don't know if you run into difficulties - however ask first! Please repeat the questions in your answer for easier marking.

(MODELLING - EQUALITIES AND INEQUALITIES)

For each of the situations below, give examples of equalities or inequalities that you could use to describe the situation quantitatively. You may use words in the formulas, like in

money after = money before – price

- Two given objects on a balance scale

- Three persons of different height
- Weights of three suitcases at airport (in a typical situation there)
- Water in two glasses

- In this game there is exactly one winner! (hint: use one binary variable for each person, 1 if the person wins, 0 otherwise)
- (voluntary) Loss of energy through window (try to guess a relationship if you don't know)

Consider every question, but you may answer don't know if you run into difficulties - however ask first!

(mathematical knowledge)

(REMINDER BASIC ALGEBRA)

Here is a **reminder for basic algebra**. Generally try to make sure that you are familiar with the basic concepts. If you need help please contact a teacher.

(finally...)

(SELF-CHECK)

- Have you answered all questions to the best of your ability?
- Is all the required information on the front page, is the file name correct etc.?
- Anything else you can easily check? (details, terminology, arguments, clearly stated answers etc.?)

Do not submit an incomplete module! We are available to help you, and you can receive a short extension if you contact us.