2 Functions, equations and geometry I

Depending on your background, some of these exercises may be easy, while others may be more difficult. We simply ask you to do your best with these problems, within the limits of the time allocated for the course.

Read **these reminders** before you begin!

(keeping track)

(SIMPLE FORECAST)

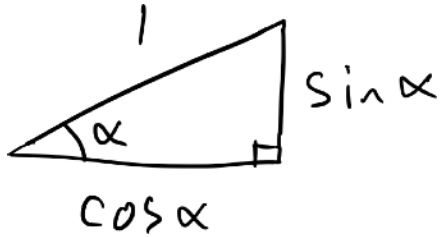
In a city, the number of daily bicyclists have been estimated every third year, see the table.

2009	24 300
2012	26 800
2015	29 100

a) The city needs an estimate for 2013, to compare with other data for 2013. Suggest an estimate for 2013 and explain how you were thinking to find it.

b) For the planning project “CITY 2020”, the city also wishes an estimate for the year 2020. Propose such an estimate, explain your thinking, and propose what you want to say to anyone who will use your estimate.

(HOMING)



When you approach land in a ship, you can establish your position on the nautical chart by using a compass to measure the angles to two different landmarks with a known position. This is a classical way to ensure a safe approach.

See this **help slide**!

a) Explain how the position of the ship can be calculated analytically, in sufficient detail so that it can easily be implemented in a computer program.

b) (for DIT856) Write a simple Python program that implements your solution.

c) (voluntary) What happens if you have three or more landmarks?

(investigating the abstract)

(INVESTIGATE MATHEMATICAL FUNCTIONS THROUGH CHANGE)

Have a look at the program **function.py**. It creates some well-known mathematical functions in a simple way, by generating them from left to right based on their rate of change (the derivative). If you cannot run yourself, here is **the output**.

a) Carefully study the mechanism of the program and try to understand it. Give some comments on the mathematical aspects of the program.

b) If you have the graph of a (one-variable) function, how can you geometrically construct the inverse of the function? Are any of the inverses of the functions shown by the program familiar to you?

(I ran the program with an Anaconda installation of Python 3 on my Mac. It needs the `matplotlib` library.)

(TWELVE BALLS PROBLEM)

This is a classical mathematical puzzle. You have twelve balls with identical appearance. However, one of the balls is slightly lighter or heavier than the others. You also have a balance scale. How many weighings do you need to find the odd ball, and determining if it is lighter or heavier. Explain how. HINT: make a serious attempt, and report the best solution you found in a limited time.

(investigating the world)

(IDENTIFY FUNCTIONS IN THE REAL WORLD)

Have a look at the different tables and diagrams (see separate page at the end), and try to link to mathematical functions you know and/or saw in the previous exercise.

Give some comments.

(SIZE OF THE EARTH)

Almost 2300 years ago, Erathostenes estimated the size of the Earth in the following way. He had heard that on the day of summer solstice the sun lit the bottom of the wells in the city of Syene, 800 km south of his home city Alexandria. On the same day, Erathostenes measured the angle to the sun to be 7 degrees and 12 minutes (a minute is a 1/60 of a degree). Figure out and explain his calculation.

(design)

(RENEWABLE ELECTRIC ENERGY SYSTEM)

Suggest a cost-aware design for a simple renewable electricity system for a small holiday cottage in the countryside, not connected to the electric grid. Motivate your design. Briefly comment also on any important practical considerations.

Some information:

- Smaller solar panels cost about 2000 SEK per 100W, and a solar charge regulator about 1000 SEK.
- **Solar panel yield in Sweden**.
- **Wind generators** (they also provide some energy yield estimates for each product)
- Batteries cost about 2000 SEK for a good 12V 100Ah battery. You should normally use only half of the capacity or the battery will more quickly degrade over time.
- A good 12V refrigerator will need about 1A.
- Obviously don't search for ready-made cottage packages for sale, the task is for you to suggest the design. However if you wish to look up other data that you find relevant you may do so.

Make sure that you end with an *unambiguous* instruction for how to dimension the system, according to your thinking.

HINT even if you feel that you lack some facts, you can either try to work without assuming anything (more complicated), or simply make some *estimates and assumptions* for example about how much energy you need, about the weather etc. (easier), and continue with the design. Details and numbers can always be changed afterwards, and continuing with the analysis still provides a lot of insight.

(thinking)

(DIFFERENT LEVELS OF UNDERSTANDING)

Are there any differences in how deeply you understand the following statements. We are asking about *your* personal understanding! For example, do you understand *what* the statement says or not? Do you also understand *why* the statement is true?

- Two parallel lines never meet
- The order of multiplication doesn't matter
- There are infinitely many prime numbers.
- The average is

$$A = \frac{1}{n} \sum_{i=1}^{n} x_i$$

- You can calculate W with the formula W=C*m ^ 2 where m is the mass and C= 0.00236481
- The formula

$$\operatorname{div}\left(\operatorname{curl} \vec{F}\right) = \frac{\partial}{\partial x}(z) + \frac{\partial}{\partial y}(2yz) + \frac{\partial}{\partial z}(y - z^2) = 2z - 2z = 0$$

Summarize your thoughts. Please write out any statements you refer to for easier marking.

(MATHEMATICAL THINKING LECTURE)

Did you find anything particularly relevant for you in the mathematical thinking lecture? (last Thursday - see the lecture notes).

(mathematical knowledge)

(EUCLID'S ELEMENTS)

Euclid's Elements, written about 2300 years ago, is one of the most successful textbooks of all time. Browse the book and give some brief impressions! What do you think the purpose of the book is? How is the mathematical knowledge presented? What are the similarities and differences to a modern-day maths textbook? (You can find the proof of Thales' theorem in Chapter 3, proposition 31. Pythagoras' theorem is in Book 1, proposition 47. The later parts with number theory are also interesting.)

Web version of Euclid's elements

Typeset version of Euclid's elements

(REMINDER FUNCTIONS AND EQUATIONS)

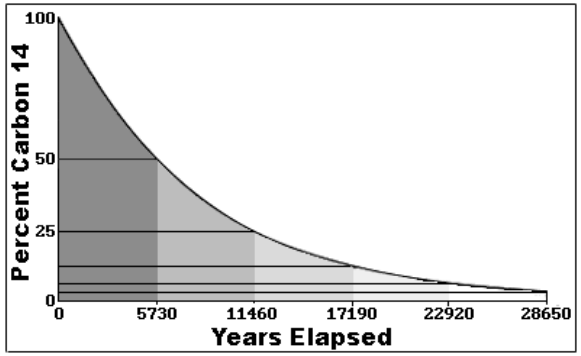
Have a look at the introductory lecture notes for this module, as well as the more extensive lecture notes which are written in a more classical mathematical style. Generally try to make sure that you are familiar with the basic concepts. If you need help please contact a teacher.

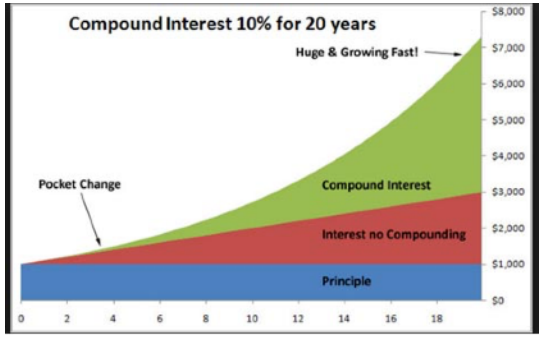
(finally...)

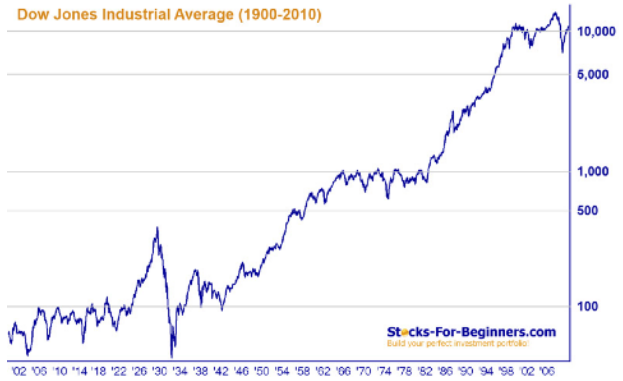
(SELF-CHECK)

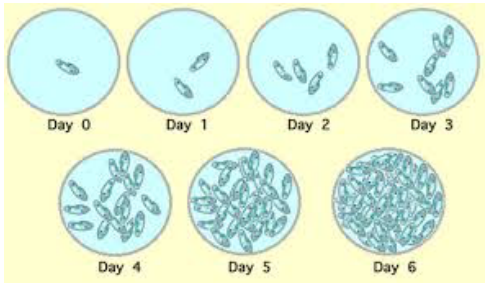
- Have you answered all questions to the best of your ability?
- Is all the required information on the front page, is the file name correct etc.?
- Anything else you can easily check? (details, terminology, arguments, clearly stated answers etc.?)

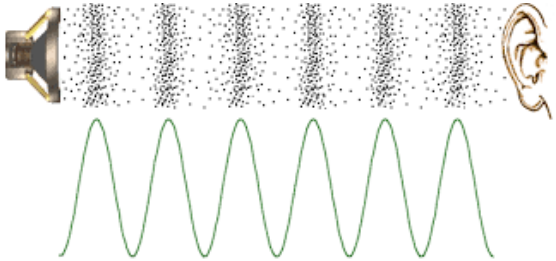
Do not submit an incomplete module! We are available to help you, and you can receive a short extension if you contact us.











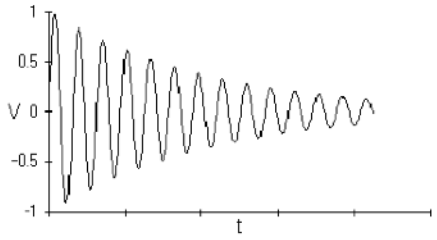


Figure 5
The voltage signal induced
in the receiver coil versus time.

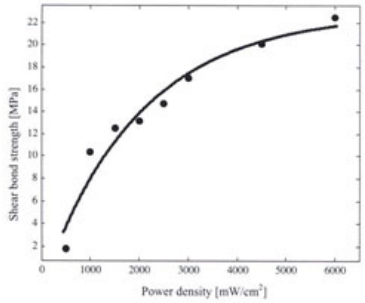


Fig. 1 – Shear bond strength as a function of power density. An exponential model (see Table 4) describes the relationship between shear bond strength and power density. The best-fit curve based on this model shows that shear bond strength enters a region of saturation at a power density of approximately 4000 mW/cm².